

# Model-independent constraint on the neutrino mass spectrum from the Neutrinoless Double Beta Decay

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## Abstract

We present a concise formula to relate the effective mass term of the neutrinoless double beta decay to a single neutrino mass, two Majorana phases and some observables of neutrino oscillations for four possible patterns of the neutrino mass spectrum: (1)  $m_1 > m_2 > m_3$ ; (2)  $m_1 < m_2 < m_3$ ; (3)  $m_1 < m_2 > m_3$ ; and (4)  $m_1 > m_2 < m_3$ . If the alleged evidence for the neutrinoless double beta decay is taken into account, one may obtain a rough but model-independent constraint on the absolute scale of neutrino masses:  $0.1 \text{ eV} \leq m_3 \leq 1 \text{ eV}$ . This result implies that three neutrino masses are nearly degenerate.

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## I. INTRODUCTION

The solar and atmospheric neutrino oscillations observed in the Super-Kamiokande experiment [1] have provided convincing evidence that neutrinos are massive and lepton flavors are mixed. If neutrinos are Majorana particles, a complete description of the flavor mixing phenomenon in the framework of three lepton families requires six real parameters: three mixing angles, one Dirac-type CP-violating phase and two Majorana-type CP-violating phases. So far some preliminary knowledge on three flavor mixing angles and two neutrino mass-squared differences have been achieved from current neutrino oscillation experiments. It is likely to determine the Dirac-type CP-violating phase from a new generation of accelerator neutrino experiments with very long baselines, if the solar neutrino anomaly is attributed to the large-angle Mikheyev-Smirnov-Wolfenstein (MSW) oscillation [2] and the flavor mixing angle between the first and third lepton families is not too small. To pin down two Majorana phases is practically impossible, however, since all possible lepton-number-nonconserving processes induced by light Majorana neutrinos are suppressed in magnitude by extremely small factors compared to normal weak interactions [3]. The only experimental possibility to get some information on two Majorana-type CP-violating phases is to measure the neutrinoless double beta decay.

Recently Klapdor-Kleingrothaus *et al* have reported their first evidence for the existence of the neutrinoless double beta decay [4]. At the 95% confidence level, the effective mass term of the neutrinoless double beta decay is found to lie in the following range:

$$0.05 \text{ eV} \leq \langle m \rangle_{ee} \leq 0.84 \text{ eV} . \quad (1)$$

A number of authors have discussed the implications of this alleged evidence on neutrino masses [5] and textures of the neutrino mass matrix [6–11].

The purpose of this paper is two-fold. First, we present a concise formula to relate  $\langle m \rangle_{ee}$  to a single neutrino mass, two Majorana phases and some observables of neutrino oscillations for four possible patterns of the neutrino mass spectrum: (1)  $m_1 > m_2 > m_3$ ; (2)  $m_1 < m_2 < m_3$ ; (3)  $m_1 < m_2 > m_3$ ; and (4)  $m_1 > m_2 < m_3$ . Second, we take the experimental result in Eq. (1) seriously and obtain a rough but model-independent constraint on the absolute scale of neutrino masses:  $0.1 \text{ eV} \leq m_3 \leq 1 \text{ eV}$ . This result implies that three neutrino masses are nearly degenerate.

## II. FORMULATION

Current experimental data [1,12] indicate that solar and atmospheric neutrino oscillations are dominated by  $\nu_e \rightarrow \nu_\mu$  and  $\nu_\mu \rightarrow \nu_\tau$  transitions, respectively. The neutrino mass-squared differences associated with solar and atmospheric neutrino oscillations are thus defined as

$$\begin{aligned} \Delta m_{\text{sun}}^2 &\equiv \left| m_2^2 - m_1^2 \right| , \\ \Delta m_{\text{atm}}^2 &\equiv \left| m_3^2 - m_2^2 \right| , \end{aligned} \quad (2)$$

where  $m_i$  (for  $i = 1, 2, 3$ ) denote the physical (real and positive) masses of electron, muon and tau neutrinos. The observed hierarchy between  $\Delta m_{\text{sun}}^2$  and  $\Delta m_{\text{atm}}^2$  can tell the relative

sizes of three neutrino masses, but it cannot shed any light on the absolute value of  $m_1$ ,  $m_2$  or  $m_3$ . In order to show how the absolute scale of neutrino masses can be constrained from the neutrinoless double beta decay, we express  $m_1$  and  $m_2$  in terms of  $m_3$ ,  $\Delta m_{\text{sun}}^2$  and  $\Delta m_{\text{atm}}^2$  with the help of Eq. (2). The results are summarized as

$$\begin{aligned} m_1 &= \sqrt{m_3^2 + p\Delta m_{\text{atm}}^2 + q\Delta m_{\text{sun}}^2} , \\ m_2 &= \sqrt{m_3^2 + p\Delta m_{\text{atm}}^2} , \end{aligned} \quad (3)$$

where  $p = \pm 1$  and  $q = \pm 1$  stand for four possible patterns of the neutrino mass spectrum:

$$(p, q) = (+1, +1) : \quad m_1 > m_2 > m_3 ; \quad (4a)$$

$$(p, q) = (-1, -1) : \quad m_1 < m_2 < m_3 ; \quad (4b)$$

$$(p, q) = (+1, -1) : \quad m_1 < m_2 > m_3 ; \quad (4c)$$

$$(p, q) = (-1, +1) : \quad m_1 > m_2 < m_3 . \quad (4d)$$

We see that  $m_1 \approx m_2$  holds as a straightforward consequence of  $\Delta m_{\text{sun}}^2 \ll \Delta m_{\text{atm}}^2$ . The signs of  $p$  and  $q$  can be determined from the future long-baseline neutrino oscillation experiments with high-quality conventional neutrino beams [13] or at neutrino factories [14].

As solar and atmospheric neutrino oscillations are approximately decoupled from each other, their mixing factors  $\sin^2 2\theta_{\text{sun}}$  and  $\sin^2 2\theta_{\text{atm}}$  may have simple relations with the matrix elements of the lepton flavor mixing matrix  $V$ , which is defined to link the neutrino mass eigenstates  $(\nu_1, \nu_2, \nu_3)$  to the neutrino flavor eigenstates  $(\nu_e, \nu_\mu, \nu_\tau)$ :

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} . \quad (5)$$

The mixing factor associated with the CHOOZ (or Palo Verde) reactor neutrino oscillation experiment [15], denoted as  $\sin^2 2\theta_{\text{chz}}$ , is also a simple function of the matrix elements of  $V$  in the same approximation. The explicit expressions of  $\sin^2 2\theta_{\text{sun}}$ ,  $\sin^2 2\theta_{\text{atm}}$  and  $\sin^2 2\theta_{\text{chz}}$  read as follows:

$$\begin{aligned} \sin^2 2\theta_{\text{sun}} &= 4|V_{e1}|^2|V_{e2}|^2 , \\ \sin^2 2\theta_{\text{atm}} &= 4|V_{\mu 3}|^2 (1 - |V_{\mu 3}|^2) , \\ \sin^2 2\theta_{\text{chz}} &= 4|V_{e3}|^2 (1 - |V_{e3}|^2) . \end{aligned} \quad (6)$$

Taking the unitarity of  $V$  into account, one may reversely express  $|V_{e1}|^2$ ,  $|V_{e2}|^2$ ,  $|V_{e3}|^2$  and  $|V_{\mu 3}|^2$  in terms of  $\theta_{\text{sun}}$ ,  $\theta_{\text{atm}}$  and  $\theta_{\text{chz}}$ :

$$\begin{aligned} |V_{e1}|^2 &= \frac{1}{2} \left( \cos^2 \theta_{\text{chz}} + \sqrt{\cos^4 \theta_{\text{chz}} - \sin^2 2\theta_{\text{sun}}} \right) , \\ |V_{e2}|^2 &= \frac{1}{2} \left( \cos^2 \theta_{\text{chz}} - \sqrt{\cos^4 \theta_{\text{chz}} - \sin^2 2\theta_{\text{sun}}} \right) , \\ |V_{e3}|^2 &= \sin^2 \theta_{\text{chz}} , \\ |V_{\mu 3}|^2 &= \sin^2 \theta_{\text{atm}} . \end{aligned} \quad (7)$$

Current experimental data favor  $\theta_{\text{chz}} \ll 1$  and  $\theta_{\text{sum}} \sim \theta_{\text{atm}} \sim 1$ , therefore  $|V_{e1}|^2 \sim |V_{e2}|^2 \sim |V_{\mu 3}|^2 \gg |V_{e3}|^2$  is expected to hold.

Note that only the matrix elements  $V_{e1}$ ,  $V_{e2}$  and  $V_{e3}$  are relevant to the neutrinoless double beta decay. Without loss of generality, one may redefine the phases of three charged lepton fields in an appropriate way such that the phases of  $V_{e1}$  and  $V_{e2}$  are purely of the Majorana type and  $V_{e3}$  is real [16]. In other words,

$$\arg(V_{e1}) = \rho, \quad \arg(V_{e2}) = \sigma, \quad \arg(V_{e3}) = 0. \quad (8)$$

Of course  $\rho$  and  $\sigma$  do not have any effect on CP or T violation in normal neutrino-neutrino and antineutrino-antineutrino oscillations [17]. With the help of Eqs. (3), (7) and (8), we then arrive at a model-independent expression for the effective mass term of the neutrinoless double beta decay:

$$\begin{aligned} \langle m \rangle_{ee} &= \left| m_1 V_{e1}^2 + m_2 V_{e2}^2 + m_3 V_{e3}^2 \right| \\ &= \left| m_3 \sin^2 \theta_{\text{chz}} + \frac{\cos^2 \theta_{\text{chz}}}{2} \left( \sqrt{m_3^2 + p \Delta m_{\text{atm}}^2 + q \Delta m_{\text{sun}}^2} e^{2i\rho} + \sqrt{m_3^2 + p \Delta m_{\text{atm}}^2} e^{2i\sigma} \right) \right. \\ &\quad \left. + \frac{\sqrt{\cos^4 \theta_{\text{chz}} - \sin^2 2\theta_{\text{sun}}}}{2} \left( \sqrt{m_3^2 + p \Delta m_{\text{atm}}^2 + q \Delta m_{\text{sun}}^2} e^{2i\rho} - \sqrt{m_3^2 + p \Delta m_{\text{atm}}^2} e^{2i\sigma} \right) \right|. \quad (9) \end{aligned}$$

One can see that  $\langle m \rangle_{ee}$  consists of three unknown parameters:  $m_3$ ,  $\rho$  and  $\sigma$ , which are unable to be determined from any neutrino oscillation experiments. Once  $\Delta m_{\text{sun}}^2$ ,  $\Delta m_{\text{atm}}^2$ ,  $\theta_{\text{sun}}$ ,  $\theta_{\text{atm}}$  and  $\theta_{\text{chz}}$  are measured to an acceptable degree of accuracy, we will be able to get a useful constraint on the absolute neutrino mass  $m_3$  for arbitrary values of  $\rho$  and  $\sigma$  from the observation of  $\langle m \rangle_{ee}$ . If the magnitude of  $m_3$  could roughly be known from some cosmological constraints, it would be likely to obtain some loose but instructive information on the Majorana phases  $\rho$  and  $\sigma$  by confronting Eq. (9) with the experimental result of  $\langle m \rangle_{ee}$ . Anyway further progress in our theoretical understanding of the origin of neutrino masses and CP violation is crucial for a complete determination of the free parameters under discussion.

### III. ILLUSTRATION

Now let us illustrate the dependence of  $\langle m \rangle_{ee}$  on  $m_3$ ,  $\rho$  and  $\sigma$  numerically. Assuming that the solar neutrino anomaly is attributed to the large-angle MSW effect [2], we typically take  $\Delta m_{\text{sun}}^2 = 5 \cdot 10^{-5} \text{ eV}^2$  and  $\sin^2 2\theta_{\text{sun}} = 0.8$ . We choose  $\Delta m_{\text{atm}}^2 = 3 \cdot 10^{-3} \text{ eV}^2$  and  $\sin^2 2\theta_{\text{atm}} = 1$  for the atmospheric neutrino oscillation. In addition, we use the typical value  $\sin^2 2\theta_{\text{chz}} = 0.05$  in our numerical calculations, which is consistent with the upper bound  $\sin^2 2\theta_{\text{chz}} < 0.1$  from the CHOOZ reactor neutrino experiment [15]<sup>1</sup>. The Majorana phases  $\rho$  and  $\sigma$  are completely unknown. To illustrate, we consider four instructive possibilities for

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<sup>1</sup>Note that  $\sin^2 2\theta_{\text{chz}} \sim 0.05$  is also favored in a number of phenomenological models of lepton mass matrices. See Ref. [18] for a review with extensive references.

$\rho$  and  $\sigma$ : (1)  $\rho = \sigma = 0$ ; (2)  $\rho = \pi/4$  and  $\sigma = 0$ ; (3)  $\rho = 0$  and  $\sigma = \pi/4$ ; and (4)  $\rho = \sigma = \pi/4$ . Our results for  $\langle m \rangle_{ee}$  as a function of  $m_3$  are shown in FIGs. 1–4, corresponding to four possible patterns of the neutrino mass spectrum listed in Eq. (4). Some comments are in order.

(1) Comparing between FIG. 1 and FIG. 3 (or FIG. 2 and FIG. 4), we see that the result of  $\langle m \rangle_{ee}$  is essentially insensitive to the sign of  $q$ . In other words, the cases  $m_1 > m_2$  and  $m_1 < m_2$  are almost indistinguishable in the neutrinoless double beta decay. This feature is a straightforward consequence of the hierarchy  $\Delta m_{\text{sun}}^2 \ll \Delta m_{\text{atm}}^2$ . As shown in Eq. (9), the contribution of  $q\Delta m_{\text{sun}}^2$  to  $\langle m \rangle_{ee}$  is negligible unless a complete cancellation between  $m_3^2$  and  $p\Delta m_{\text{atm}}^2$  terms happens to take place.

(2) When  $p = -1$  (i.e.,  $m_2 < m_3$ ), Eq. (3) implies that  $m_3$  has the following lower bound

$$m_3 \geq \begin{cases} \sqrt{\Delta m_{\text{atm}}^2} & (q = +1) , \\ \sqrt{\Delta m_{\text{atm}}^2 + \Delta m_{\text{sun}}^2} & (q = -1) . \end{cases} \quad (10)$$

Considering the typical inputs of  $\Delta m_{\text{atm}}^2$  and  $\Delta m_{\text{sun}}^2$  taken above, we obtain  $m_3 \geq 0.0548$  eV for  $q = +1$  and  $m_3 \geq 0.0552$  eV for  $q = -1$ . Such lower bounds of  $m_3$  have been reflected in FIGs. 2 and 4.

(3) We find that it is numerically difficult to distinguish between the possibilities  $\rho = \sigma = 0$  and  $\rho = \sigma = \pi/4$ . The reason is simply that the second term on the right-hand side of Eq. (9) dominates over the other two terms, when  $\rho = \sigma$  is taken. Hence the value of  $\langle m \rangle_{ee}$  becomes insensitive to the explicit values of two identical Majorana phases. One can also see that the possibility of  $\rho = 0$  and  $\sigma = \pi/4$  is almost indistinguishable from the possibility of  $\rho = \pi/4$  and  $\sigma = 0$ . In this specific case the first term on the right-hand side of Eq. (9) plays an insignificant role, therefore the magnitude of  $\langle m \rangle_{ee}$  is essentially invariant under an exchange of the values between  $\rho$  and  $\sigma$ .

(4) We observe that the changes of  $\langle m \rangle_{ee}$  are rather mild for the typical values of  $\rho$  and  $\sigma$  chosen above. If reasonable inputs of  $\sin^2 2\theta_{\text{sun}}$ ,  $\sin^2 2\theta_{\text{atm}}$  and  $\sin^2 2\theta_{\text{chz}}$  are taken <sup>2</sup>, a careful numerical scan shows that the magnitude of  $\langle m \rangle_{ee}$  does not undergo any dramatic changes for arbitrary  $\rho$  and  $\sigma$ . Thus a rough but model-independent constraint on the absolute scale of neutrino masses can be obtained from the observation of  $\langle m \rangle_{ee}$ . In view of the alleged experimental region of  $\langle m \rangle_{ee}$  in Eq. (1), we find that  $m_3$  is most likely to lie in the range  $0.1 \text{ eV} \leq m_3 \leq 1 \text{ eV}$  (see FIGs. 1–4). This result is irrelevant to the details of four possible patterns of the neutrino mass spectrum. Note that  $m_3 \geq 0.1 \text{ eV}$  implies that both  $m_1 \approx m_3$  and  $m_2 \approx m_3$  hold, as one can see from Eq. (3). Therefore three neutrino masses are nearly degenerate. Taking  $m_3 = 0.5 \text{ eV}$  for example, we obtain  $m_1 + m_2 + m_3 \approx 3m_3 \approx 1.5 \text{ eV}$ . Such a sum of three neutrino masses can be translated in cosmology to  $\Omega_\nu h^2 \approx 0.016$ , where  $\Omega_\nu$  is the fraction of the critical density contributed by neutrinos and  $h$  is the dimensionless Hubble constant. This typical result is consistent with  $\Omega_\nu h^2 \approx 0.05$  [19], extracted from the CMB measurements and galaxy cluster surveys.

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<sup>2</sup>Only in a few extreme cases (e.g.,  $\cos^4 \theta_{\text{chz}} \approx \sin^2 2\theta_{\text{sun}}$  and  $\rho \approx -\sigma$ ), which seem quite unlikely, large cancellations may take place in  $\langle m \rangle_{ee}$  and make its magnitude significantly suppressed.

## IV. CONCLUSION

We have presented a concise formula to relate the effective mass term of the neutrinoless double beta decay to a single neutrino mass, two Majorana phases and some observables of neutrino oscillations for four possible patterns of the neutrino mass spectrum. Taking into account the alleged evidence for the neutrinoless double beta decay, we have obtained a rough but model-independent constraint on the absolute scale of active neutrino masses:  $0.1 \text{ eV} \leq m_3 \leq 1 \text{ eV}$ . This result implies that three neutrino masses are nearly degenerate.

If the existence of the neutrinoless double beta decay can be confirmed, it will be desirable to build new phenomenological models of lepton mass matrices which can accommodate three neutrinos of nearly degenerate masses and reflect their Majorana nature. Further studies of various lepton-number-violating processes will also become more realistic and important.

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*Note added:* When this paper was being completed, the preprint of Minakata and Sugiyama [20] appeared. Their best-fit analysis leads to  $0.11 \text{ eV} \leq \langle m \rangle_\beta \leq 1.3 \text{ eV}$  for the mass parameter in the single beta decay experiments. This result is consistent with ours for the absolute scale of neutrino masses.

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# FIGURES

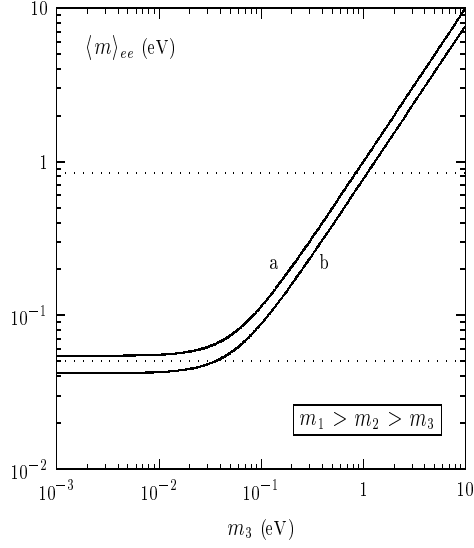


FIG. 1. Dependence of  $\langle m \rangle_{ee}$  on  $m_3$  for the neutrino mass spectrum  $m_1 > m_2 > m_3$  (the solid curves): (a)  $\rho = \sigma = 0$  or  $\pi/4$ ; and (b)  $\rho = 0$  (or  $\pi/4$ ) and  $\sigma = \pi/4$  (or  $0$ ). The region between two dashed lines corresponds to the experimentally allowed values of  $\langle m \rangle_{ee}$  at the 95% confidence level.



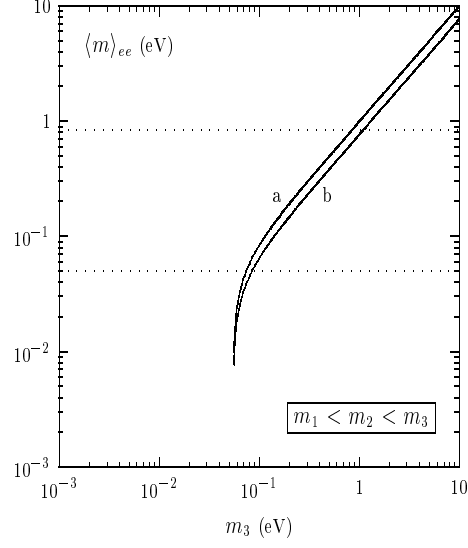


FIG. 2. Dependence of  $\langle m \rangle_{ee}$  on  $m_3$  for the neutrino mass spectrum  $m_1 < m_2 < m_3$  (the solid curves): (a)  $\rho = \sigma = 0$  or  $\pi/4$ ; and (b)  $\rho = 0$  (or  $\pi/4$ ) and  $\sigma = \pi/4$  (or  $0$ ). The region between two dashed lines corresponds to the experimentally allowed values of  $\langle m \rangle_{ee}$  at the 95% confidence level.

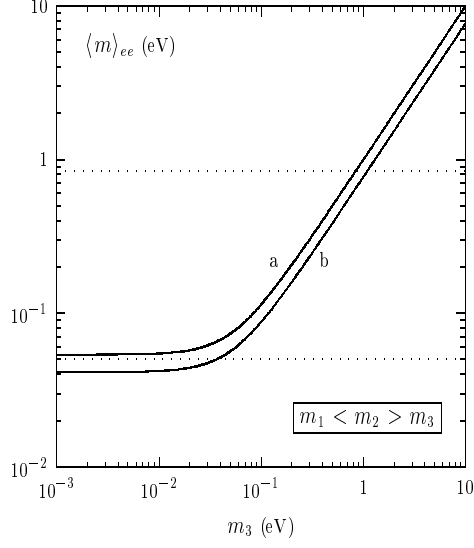


FIG. 3. Dependence of  $\langle m \rangle_{ee}$  on  $m_3$  for the neutrino mass spectrum  $m_1 < m_2 > m_3$  (the solid curves): (a)  $\rho = \sigma = 0$  or  $\pi/4$ ; and (b)  $\rho = 0$  (or  $\pi/4$ ) and  $\sigma = \pi/4$  (or  $0$ ). The region between two dashed lines corresponds to the experimentally allowed values of  $\langle m \rangle_{ee}$  at the 95% confidence level.

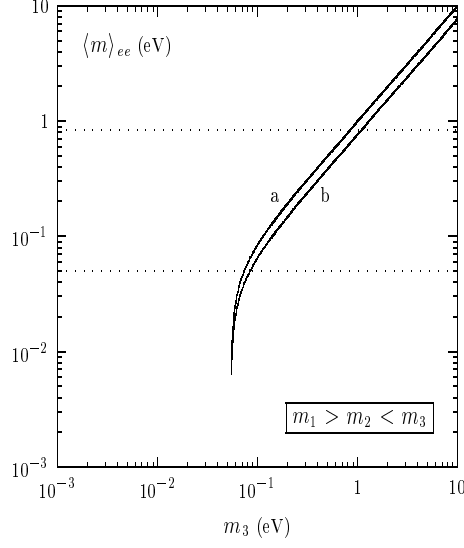


FIG. 4. Dependence of  $\langle m \rangle_{ee}$  on  $m_3$  for the neutrino mass spectrum  $m_1 > m_2 < m_3$  (the solid curves): (a)  $\rho = \sigma = 0$  or  $\pi/4$ ; and (b)  $\rho = 0$  (or  $\pi/4$ ) and  $\sigma = \pi/4$  (or  $0$ ). The region between two dashed lines corresponds to the experimentally allowed values of  $\langle m \rangle_{ee}$  at the 95% confidence level.